Quantum Interference

by Javito
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There is a different road to reach the idea of quantum, which is that of interference. As always, let's start with classical mechanics and check...

Let's consider a particle that can leave a box using different exits. All of them are equally likely, let's say. Exits 1 and 2 lead the particle, finally, to point A. For the other exits, we don't care.

Now we will assume that the probability for the particle of choosing exit 1 is, say 0.09, and that of choosing exit 2 is 0.16. What is, then, the probability of reaching point A? It's straightforward: you add the probabilities, so 0.25.

The quantum case is slightly different. Probabilities simply can't be added directly. But there is another magnitude, quite different, which you can add so as to obtain the total probability of the particle getting to a certain place. It's the *probability amplitude* ψ.

You can think of it in various ways. A 2D vector, or arrow, or a complex number. The important thing is that it contains two pieces of information. The first is the modulus $|\psi|$. When it's squared, you obtain the probability: $p = |\psi|^2$. The second is the phase, the angle.

Let's be concrete and return to our example. The probability for the particle going through exit 1 was 0.09. Therefore, the modulus of the probability amplitude $|\psi_1|$ was 0.3, right? But we don't know anything about the angle... About the other exit, 2, the probability was 0.16. Therefore, the modulus of the amplitude was $|\psi_2| = 0.4$. Now, what is the probability of the particle getting point A?

We don't know yet! It depends on the phases... Consider the two cases depicted in figure 1.

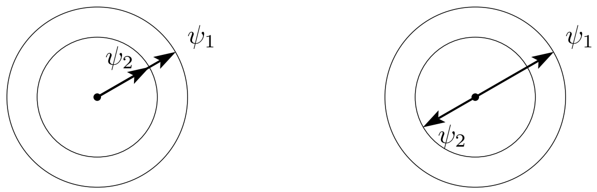

FIGURE 1. Interference in the particle getting to point A.

In the left case, the interference is constructive, and the total probability amplitude is 0.7, the probability raises to 0.49. In the second case, the interference is *destructive*, so the probability amplitude comes as low as 0.1, therefore the probability is 0.01! So, summarizing: in the classical case, probability was just 0.25. In the quantum case, it depends: you may get any result from 0.01 to 0.49.

On what does it depend, this phase? On the concrete dynamics at hand. Exactly, what happens to the particle through both paths? This is natural, right? So, in general, when you want to find the probability for any process, get the probability amplitude for all possible paths, add them up, and get the modulus squared. As easy as that. Let's try another example.

Figure 2 shows a particle that is going to collide directly with a certain target, aiming directly at the center. We will not assume the target to be symmetric in any sense, ein? But we'll consider the problem to be 2D, in order to simplify things.

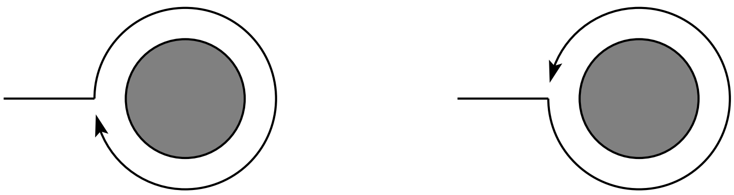

FIGURE 2. The two possible paths of the quantum particle to return to its original point.

Let's consider the probability that the particle, after the collision, returns to its initial state. I.e.: that it gets *backscattered*. There are a lot of paths which take the particle back to its initial state, but let's think of a single one, the one depicted in figure 2. It may be traveled in two different senses, right? But the two paths are *exactly* the same, only the sense is different, right? Therefore, they should get equal probability amplitudes: $\psi_1 = \psi_2 = \psi$.

If the particle were classical, then we would have to add up the probabilities: $p_c = |\psi_1|^2 + |\psi_2|^2 = 2|\psi|^2$. But what if it is quantum? Ehm... then, we have to add the amplitudes, we said: $\psi_T = \psi_1 + \psi_2 = 2\psi$. Now, square: $p_q = |2\psi|^2 = 4|\psi^2|$. Wait: this means that the probability of returning to the original point has *doubled*! Can this be true? What is the *real* difference between a classical and a quantum process?

Quoting Alice in Wonderland, *"Adventures first!"* So, let's discuss first the experiemental confirmation of this idea. Generally speaking, a metal without defects and whose nuclei remained quiet would have zero resistance. But electrons get scattered in defects, and sometimes they even rebound, i.e.: get backscattered. But the probability of getting backscattered is *double* in quantum mechanics, isn't it? So, the resistance is higher in the quantum case. This phenomenon, in 2D, is called **weak localization**.

So far, so good, but we can't check that! We can't convert the electrons into classical objects just to check that, can we? Then, what to do? A magnetic field!!

I said before that I was not going to give the rule to compute the amplitudes. I lied, I'm going to give one. If there is a magnetic field present, then the probability amplitude for a *closed path* (one that returns to its original point) gets a change in its phase. How to compute it? First we find the *magnetic flux* encircled by the trajectory, Φ.

We have to take into account the *right hand rule*, don't forget that! In practice, this means that if a trajectory has flux Φ, the trajectory which is equivalent, but traversed oppositely has flux $-\Phi$. See figure 3.

Figure 3. The probability amplitude for a closed path changes phase in $(e/\hbar)\Phi$ when there is a magnetic field present.

Hey, then if we apply a magnetic field, the two trajectories get different phases. One gets $(e/\hbar)\Phi$, and the other one gets $-(e/\hbar)\Phi$, right? Then they will not interfere constructively, the probability of backscattering reduces and the conductivity of the system will increase. That's correct. This phenomenon is called **negative magnetoresistance**. And it can be measured! It was done in 1981, by Sharvin and Sharvin.

How can you get maximum conductivity? When the probability of backscattering is zero. This is when the phase difference between the two equivalent trajectories is of π, i.e.: $2(e/\hbar)\Phi = \pi$. But if you continue increasing the magnetic field, the probability amplitudes will again have constructive interference... and you repeat the original situation if $2(e/\hbar)\Phi = 2\pi$. Therefore, you'll see an *oscillation* in the behaviour of the resistance of the material as you increase the flux, and the period of the oscillation will be $(e/\hbar)$. (OK, I'm not telling *all* the truth, but this is an introductory text, ok?)

By the way, in case you find it elsewhere, this is a manifestation of the **Aharanov-Bohm** effect, which is generally speaking the interference effects which you may get from inserting a magnetic field into a quantum problem...

But now, the fundamental question remains. Why in classical physics you add up probabilities, while in quantum physics you add up probability amplitudes? In other terms, why we have absolutely no interference effects in classical physics?

The rule is the following. If you can, somehow, find out which path did the particle go through, the interference is *lost* and you add the probabilities. This is strange, but the world seems to work like that! :) In our original problem, with exit 1 and exit 2, if you put a detector to tell you through which exit did the particle leave, then you'll get no interference. Then, adding the probabilities is ok.

Why so? First and foremost, quoting a famous prof, “I don’t know, and neither do any of my friends”. Having said that, we may introduce the concept of *coherence*. Think now of a particle as a wave. If the medium has some impurities, it may have small changes in the phase... See figure 4 to see what I mean.

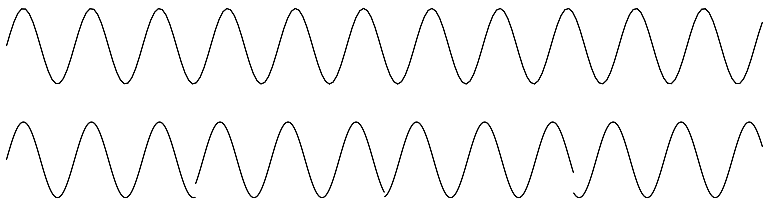

FIGURE 4. Two waves with same frequency. The lower one is losing coherence...

Figure 4 shows two waves with the same frequency, but the lower one loses coherence because of impurities or other interactions with the environment. You can see some small “breakings” of coherence. After all of them, the phase at the end is completely different than the phase at the beginning.

So, if coherence is lost, the angle between the two probability amplitudes ψ_1 and ψ_2 may be considered to be random... Thinking that any angle between 0 and π will give the same type of interference, which is the natural “bet”? Of course, $\pi/2$. But if you bet for angle $\pi/2$, you say that the arrows are orthogonal, so you may apply Pythagoras’ theorem... which means that you add the probabilities!

So, more or less, that’s the origin of the idea of quantum interference. You may apply it to more trivial examples, such as the double slit... :)