Entanglement over the rainbow: statistical mechanics of the area law

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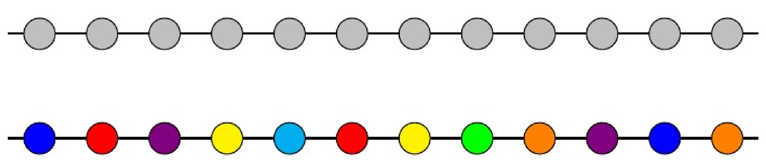
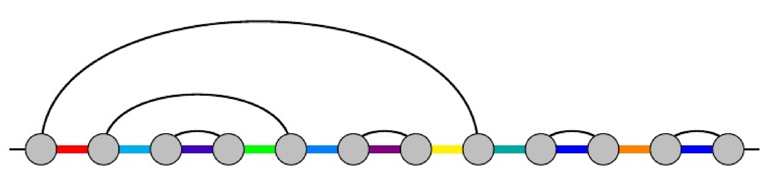
Order from disorder

Anderson's theorem:

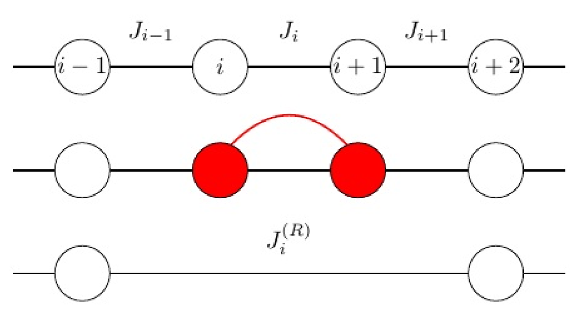
A system of 1D free electrons gets localized under any amount of uncorrelated *diagonal* disorder



If the disorder is *off-diagonal*, the system establishes random bonds



Renormalization technique of Dasgupta-Ma

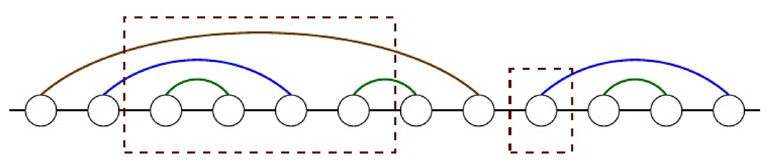


$$J_i^{(R)} = \frac{J_{i-1}J_{i+1}}{J_i}$$

if $t_i = -\log(J_i)$ then $t_i^{(R)} = t_{i-1} - t_i + t_{i+1}$

Entanglement:

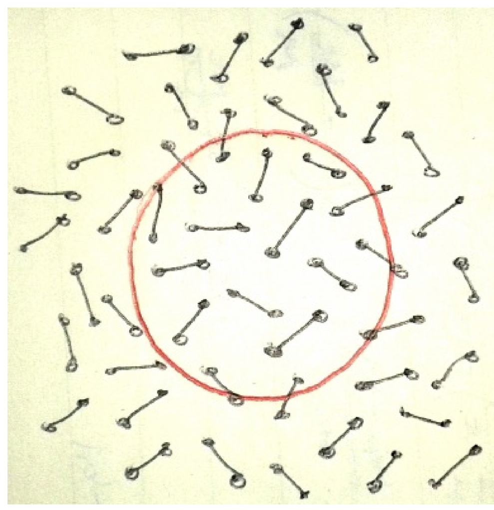
Sites connected by a bond are *entangled*: a measure on one affects the measure on the other



Entanglement entropy:
Number of bonds that are cut when a block is separated from the system

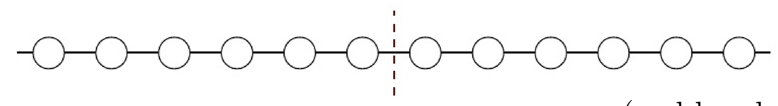
Area law:

Usually, in any quantum system at zero temperature the entanglement entropy of any block is proportional to the measure of its boundary.



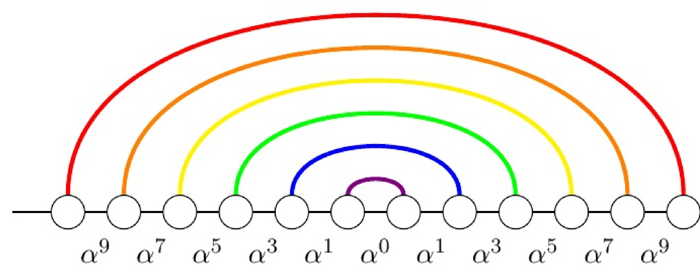
Designing quantum systems

Is it possible to create a one-dimensional system where half the sites are linked by a bond to the other half?



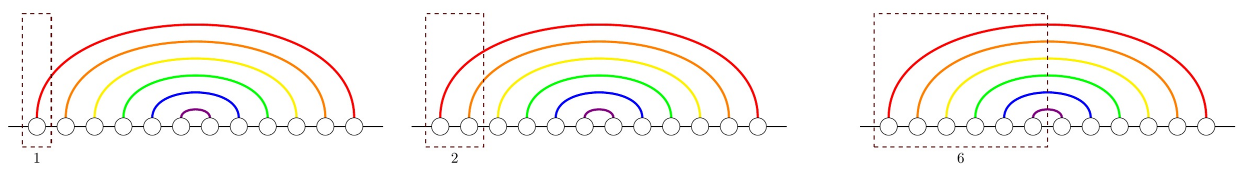
(and bonds do not cross)

The rainbow:

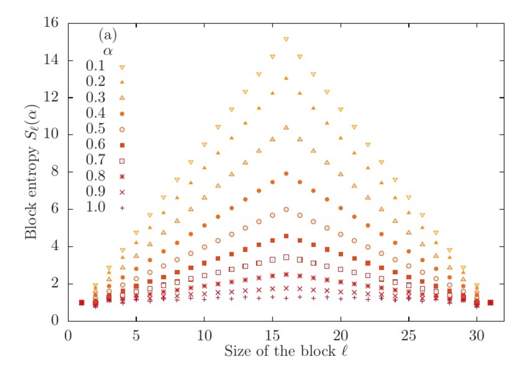


- $J = 1$ central link
- $J = \alpha^{2d-1}$ with $\alpha \in (0,1)$ and d number of sites between the link and the center

And entanglement entropy?



grows with the size of the block, but the *area* is the same



Maximum violation of area law!

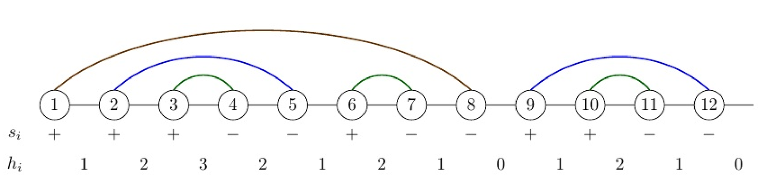
G. Ramirez et al. *J. Stat. Mech.* (2015) P06002

Let's examine other engineered systems

Random spin chains and RNA folding

Mapping between bonds and heights

$$h_i \equiv \sum_{k=1}^{i} s_k \qquad H_i \equiv h_i - h_0$$

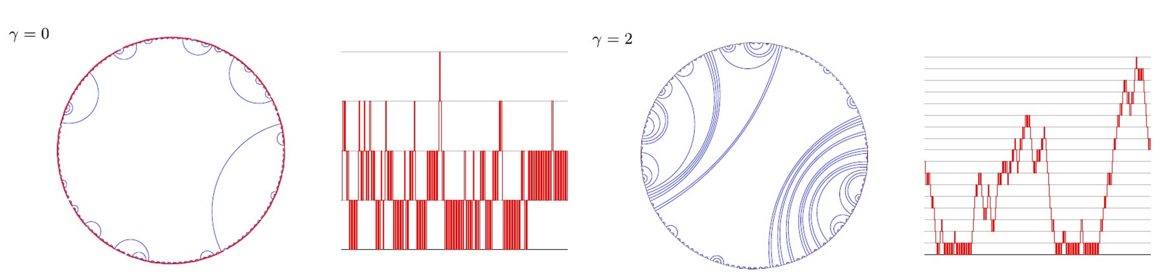


The aim is to characterize the ground states of closed spin chains when the couplings $\{J_i\}$ are random and present non-trivial long-range correlations.

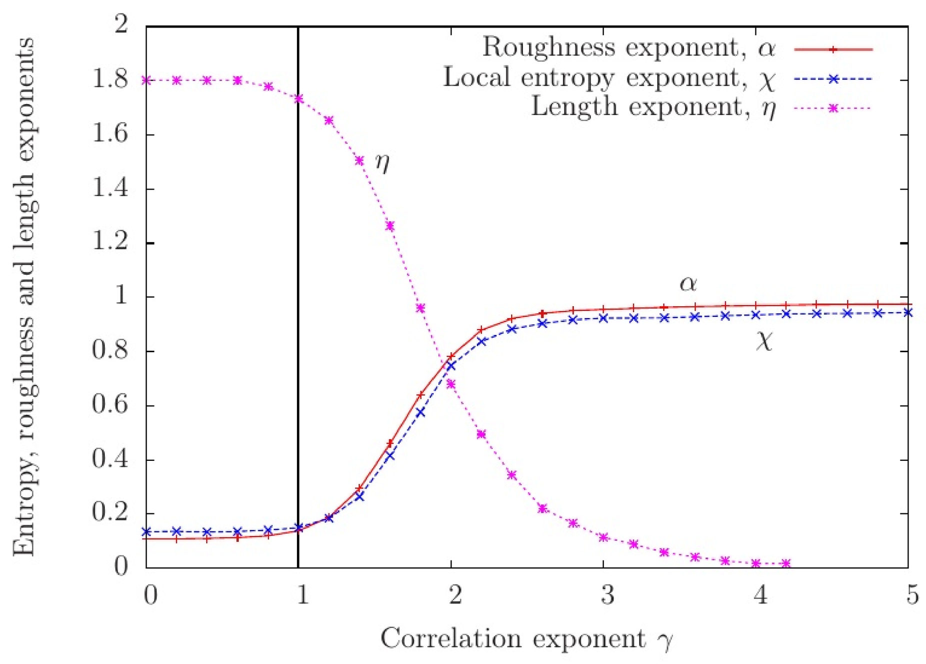
We will obtain samples from sets of log-couplings $\{t_i\}$ by employing a suitable Fourier expansion

$$t_j = \sum_k A_k \sin(jk + \phi_k)$$

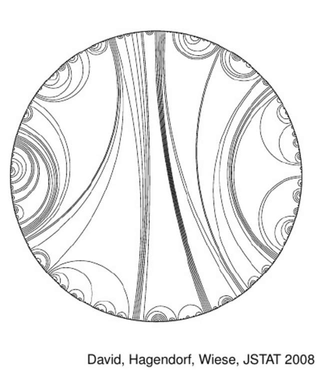
- k are a set of allowed momenta, $k_n = 2\pi n/N$, $n \in \{1, \cdots, 2N\}$
- The phase ϕ_k is taken to be uniformly distributed in $[0, 2\pi)$
- A_k is chosen as independent random variable
$$A_k = k^{-\gamma} u_k$$
u_k are independent gaussian variates, γ is a fixed spectral exponent.



- Entropy for small blocks: $S \sim \ell^{\chi}$
- Roughness as a function of the window size: $W \sim \ell^{\chi}$
- Probability distribution for the bond-length: $P \sim l^{-\eta}$



RNA folding



David, Hagendorf, Wiese, JSTAT 2008

Bond-length and entanglement scale with "dual" exponents: $\eta + \chi = 2$

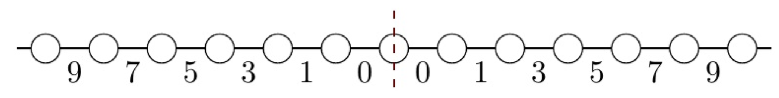
$$P(l) \approx l^{-\eta} \qquad S(\ell) \approx \ell^{\chi}$$

- RNA: $\eta = 1'44$, $\chi = 0'56$
- Random uncorrelated couplings: $\eta = 2$, $\chi = 0$.

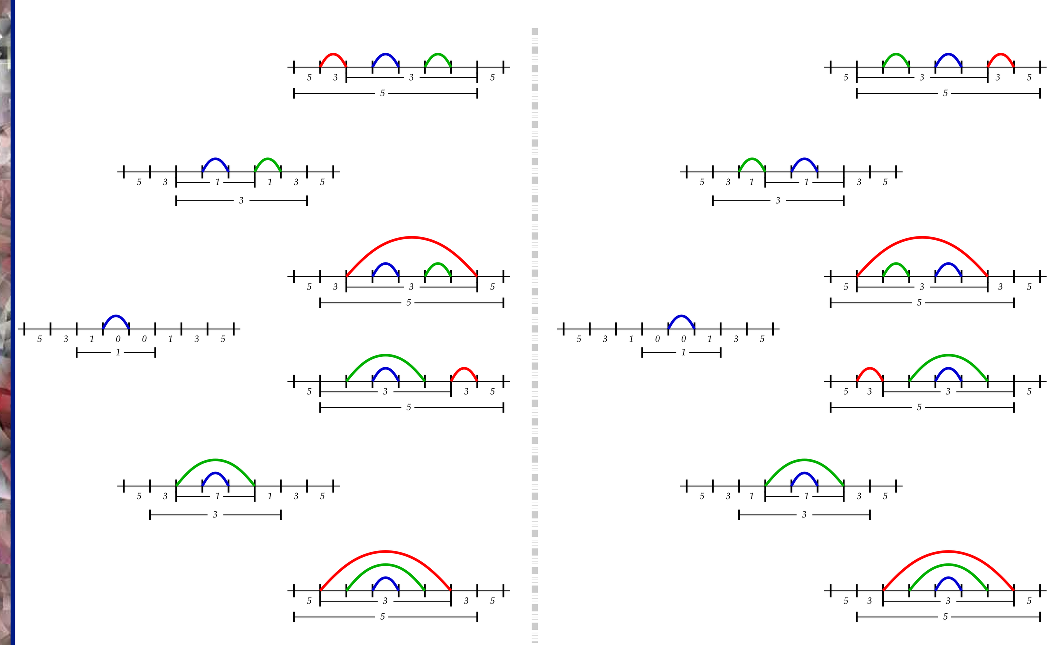
RNA folding can be simulated with spin chains!

J. Rodríguez-Laguna et al. 2016 *New J. Phys.* **18** 073025

Site-centered rainbow

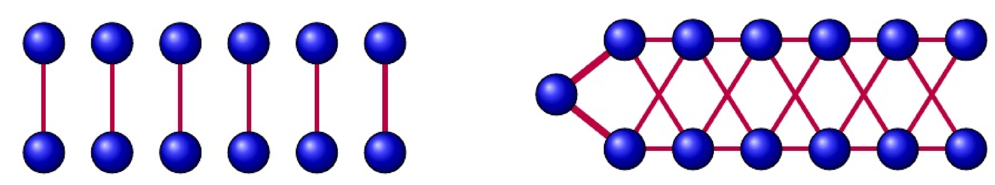


Dasgupta-Ma overexploited



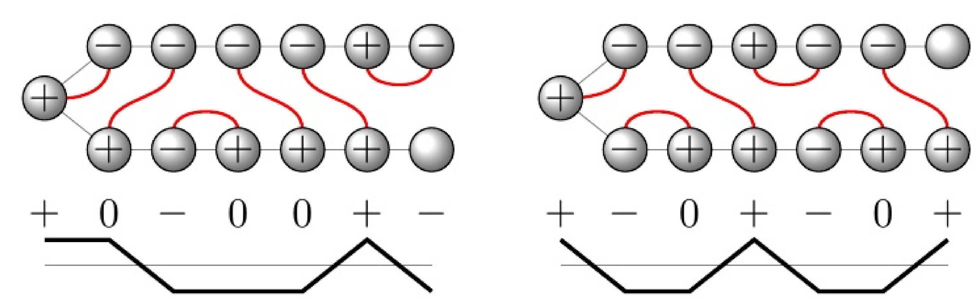
Folding the system

All effective coupling are also short-range

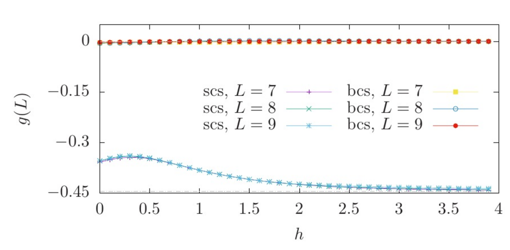


Bond-centered system and site-centered system present different topological phases protected by symmetry

Each RG step generates an effective spin 1 that couples to an effective spin 1/2 from the previous step $\Longrightarrow$ L effective spins 1 and two spins 1/2 at the ends of the folded chain. This is the AKLT state of an open chain with $L-1$ spins 1's and two 1/2's at the ends.



It can be read as a pre-rough state. Study a string order parameter $\alpha \to 0$ ($h = -\log\alpha \gg 1$)

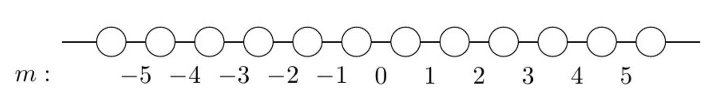


N. Samos Sáenz de Buruaga et al. arXiv:1812.04869 (accepted in *J. Stat. Mech.*)

Randbow (random-rainbow)

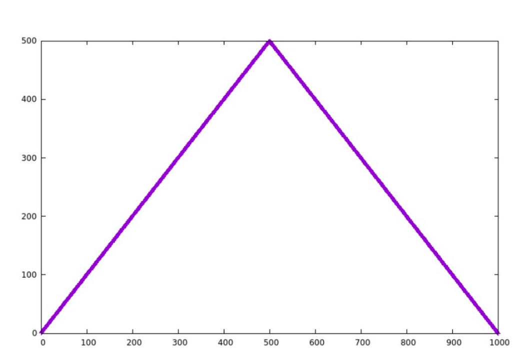
The hoppings:

$$J_m = \begin{cases} \exp\left(-h\left(\frac{1}{2} - \frac{\delta}{h}\ln\xi_0\right)\right), & m = 0, \\ \exp\left(-h\left(|m| - \frac{\delta}{h}\ln\xi_m\right)\right), & |m| > 0. \end{cases}$$

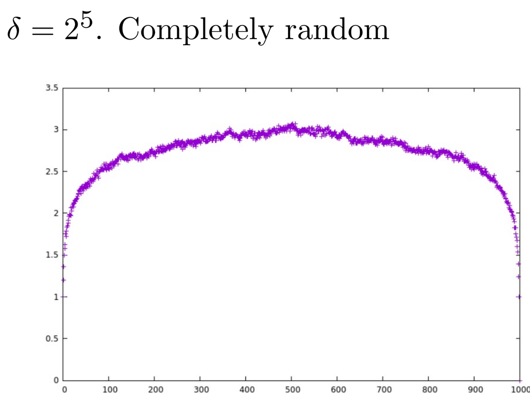


With $h > 0$ and ξ_m a random variable uniformly distributed in the interval $[0,1]$.

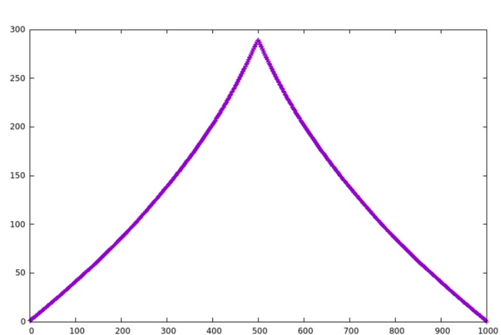
- $\delta = 0$. Rainbow

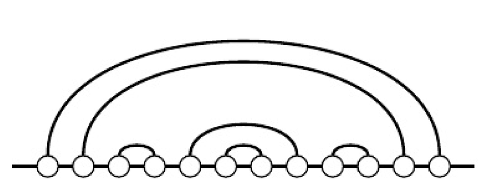


- $\delta = 2^5$. Completely random

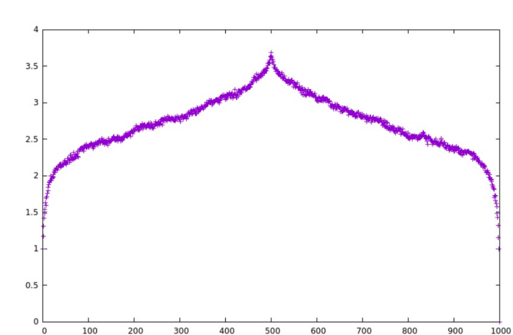


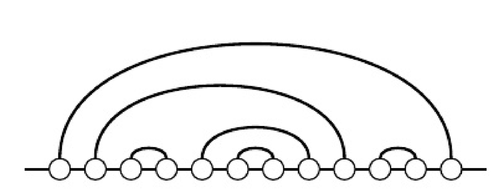
- $\delta = 2^{-4}$. Bonds are symmetric



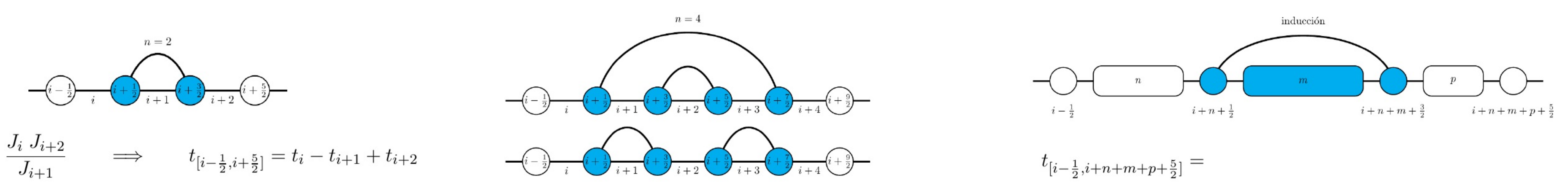


- $\delta = 2^3$. Long-range bonds remain





A useful theorem: effective hopping through many links is independent of the structure of the state. $t_{[i-1/2,i+r+1/2]} = \sum_{j=0}^{r}(-1)^j t_{i+j}$



$$J_{[i-\frac{1}{2},i+\frac{5}{2}]} = \frac{J_i J_{i+2}}{J_{i+1}} \Longrightarrow t_{[i-\frac{1}{2},i+\frac{5}{2}]} = t_i - t_{i+1} + t_{i+2}$$

$$t_{[i-\frac{1}{2},i+\frac{9}{2}]} = t_i - t_{i+1} + t_{i+2} - t_{i+3} + t_{i+4}$$

$$t_{[i-\frac{1}{2},i+n+m+p+\frac{5}{2}]} = t_{[i-\frac{1}{2},i+n+\frac{1}{2}]} - t_{[i+n+\frac{1}{2},i+n+m+\frac{3}{2}]} + t_{[i+n+m+\frac{3}{2},i+n+m+p+\frac{5}{2}]}$$

V. Alba et al. *J. Stat. Mech.* (2019) 023105